The Principles of Projective Geometry. By J. L. S. HATTON. Cambridge University Press, 1913. x + 366 pages. $3.50.

In the preface to this book the author asserts that the study of pure geometry is at present neglected and expresses the opinion that this is a misfortune. The book has been written with the hope that it may do something to alter this situation and with confidence that "the great landmarks of projective geometry" may be presented to the student in such a manner that real enthusiasm for this branch of mathematics may be aroused. The author has succeeded admirably in his attempt to write a text which gives a clear and interesting exposition of the principal methods and more important theorems of projective geometry.

The mathematical preparation with which a student is equipped when he begins the study of projective geometry is usually based on metrical concepts. Consequently the first difficulties he encounters are those arising from his being introduced all at once into an entirely new geometry from which the idea of measurement has been almost altogether excluded. The author has been aware of these difficulties and has obviated them by leading the student gradually into the non-metric geometry by the aid of metric properties. Thus the student becomes acquainted with anharmonic ratio early in the book; harmonic ratio is first defined as a special case of anharmonic ratio, and its non-metric properties and the construction of a harmonic range (pencil) from the harmonic property of a complete quadrangle (quadrilateral) are given later; the definition of involution based on its metric property is given before the non-metric definition. However, the second definition is given immediately after the first and it is proved that the metric definition might have been deduced from the second. Such a gradual transition into the study of projective geometry seems to be the best method of approach for an introductory course. The discussions and proofs are clearly presented, though enough is usually omitted to require the student to do considerable thinking for himself. The value of the book as a text is greatly enhanced by the many well-selected problems which it contains.

The first three chapters are devoted to the definitions of such fundamental concepts as range of points, pencil of lines, projection, perspective, projective relationships, anharmonic ratio and harmonic ratio. The principle of duality is here discussed and proofs of several of the properties of anharmonic and harmonic ratios, and of Ceva's and Menelaus's theorems are given. The author has used good judgment in these introductory chapters both as to the selection and the arrangement of the material.

Chapter IV contains a vivid exposition of conical (central) projection and of plane perspective. To show more clearly the differences and the similarities existing between conical projection and plane perspective, the constructions of corresponding figures and the proofs of the fundamental theorems of the two have been placed side by side.

In the next four chapters the following topics are taken up: applications of conical projection and of plane perspective to the proofs of a number of interesting theorems on triangles, quadrangles and quadrilaterals; applications of harmonic

ratio to the study of ranges and pencils in perspective, and of projective and superposed ranges and pencils; the harmonic properties of the complete quadrangle and quadrilateral; properties of involution ranges and pencils, particularly those properties of involution which are used in subsequent chapters on the conic.

In order that the matter may be more readily grasped by the student, the relations of projective forms, anharmonic properties, pole and polar, Carnot's, Pascal's and Desargues's theorems, to the circle are studied before these relations are taken up for the general conic. In the chapters on the conic some of the theorems already proved for the circle are simply restated, others are proved by methods similar to those used for circles, while in several cases alternative proofs —such as proofs by projection—are given. These chapters also contain the classification of conics and discussions of center, foci, diameters and asymptotes. The remaining eight chapters of the book deal for the most part with applications of anharmonic and harmonic ratios, involution, pole and polar, Carnot's, Pascal's and Desargues's theorems to further deductions of properties of triangles, of quadrangles and quadrilaterals, and of conics. The large number of applications and problems contained in these chapters cannot fail to interest the student of mathematics.

The addendum contains fifteen theorems and their proofs on non-projective properties of the straight line and circle. There are two very complete indexes, one of terms and definitions and the other of theorems.

The typographical errors are few and of such obvious nature as to cause the student no difficulty. The type is large and distinct and the figures have been carefully constructed.

It is the opinion of the reviewer that this book is better adapted for use as a text in a first course on projective geometry than any other book in the English language. It may also be used for reference in a lecture course on the subject. The book should do much to revive interest in the study of projective geometry in the undergraduate curriculum and "to encourage the student not to neglect the methods of pure geometry."

C. E. Stromquist.

PROBLEMS AND SOLUTIONS.

B. F. Finkel, Chairman of the Committee.

PROBLEMS FOR SOLUTION.

Special Notice.—Please reread the requests as to form of solutions on pp. 258–259 of the October 1913 issue. Unless these directions are observed by contributors, solutions must either be entirely rewritten by the committee or else rejected. Put all drawings on separate sheets.

Managing Editor.

ALGEBRA.

When this issue was made up, solutions had been received for 401, 402, 403, 404, 405, and 407. A solution of 406 is desired.